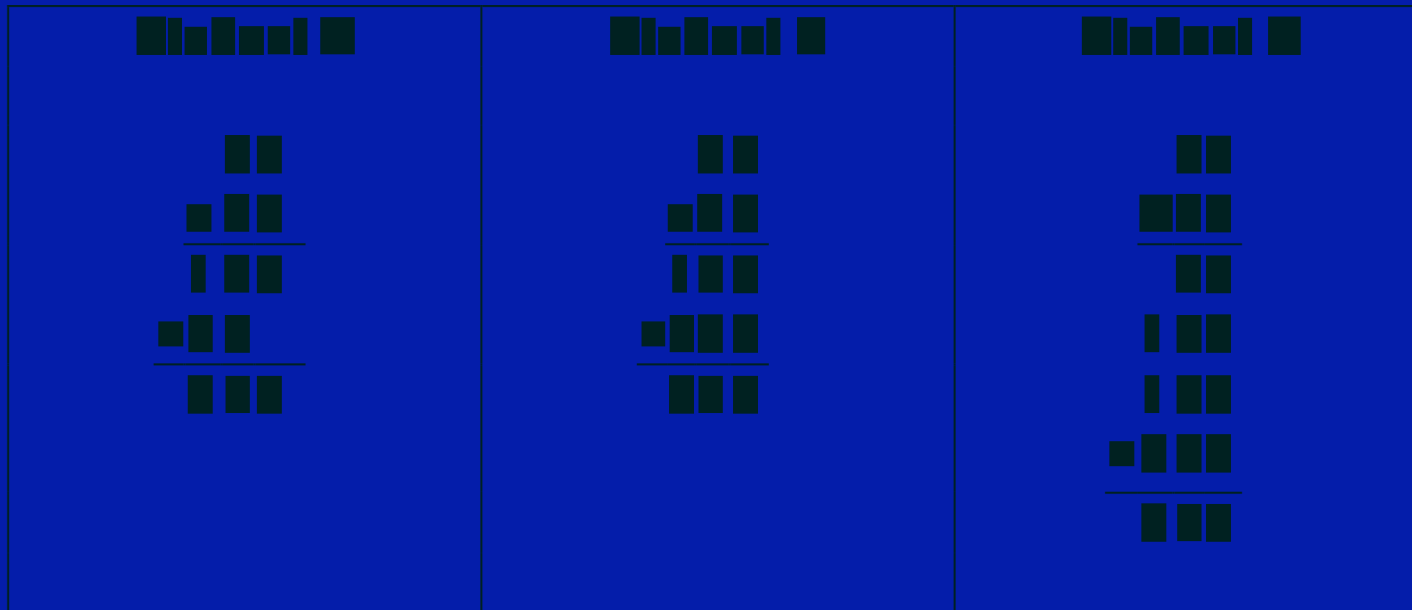


Specialized Understanding of Mathematics: A Study of Prospective Elementary Teachers

Meg Moss

What is Specialized Understanding of Mathematics (SUM)?

- Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:



Ball, D. L., Hill, H.C., Rowan, B., Schilling, S. (2002) Measuring teachers' content knowledge for teaching: Elementary mathematics release items, 2002. Ann Arbor, Michigan: Study of Instructional Improvement.

Research Questions

- 1) What are the areas of strength and what are the areas of weakness in the SUM, as measured by the Content Knowledge for Teaching Mathematics measures, of prospective elementary teachers as they enter their mathematics methods course?
- 2) Does a SUM change as prospective elementary teachers take their methods course?
- 3) What learning opportunities during the methods course may contribute to growth in SUM?

Description of Sample

- Four universities, seven sites
- n=244 pretest, n=221 posttest
- Students enrolled in elementary mathematics teaching methods course

Measures and Variables

- Content Knowledge for Teaching Mathematics – measures developed by Learning Math for Teaching/ Study for Instructional Improvement Project through The University of Michigan
- Number and Operation Content Knowledge (NOCK)
 - Common Content Knowledge
 - Specialized Content Knowledge – representing mathematical ideas, providing explanations, analyzing alternate algorithms
- Geometry Content Knowledge

SUM - Representations

7. Which of the following story problems could be used to illustrate

$1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	<u>Yes</u>	<u>No</u>	<u>I'm not sure</u>
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have \$1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3

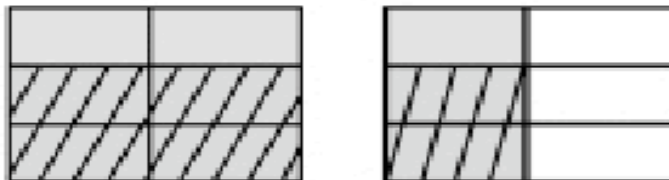
6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)

A)



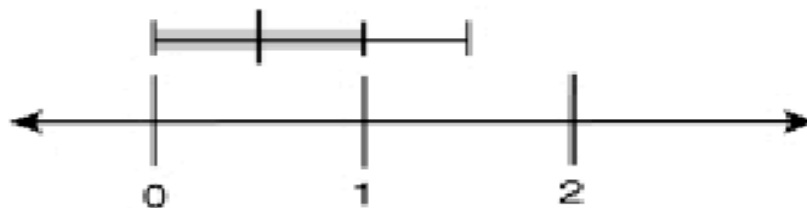
B)



C)



D)



Providing Explanations

Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4?

- a) Four is an even number, and odd numbers are not divisible by even numbers.
- b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).
- c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.
- d) It only works when the sum of the last two digits is an even number.

Methodology

- Question 1: What are the areas of strength and what are the areas of weakness in the SUM as prospective elementary teachers enter their methods course?
 - Pretest item analysis
 - Analysis of relationship between content courses and content understanding

Methodology

- Question 2: Growth during Methods Course?
 - Pretest during first two weeks of semester, posttest during last two weeks of semester
 - Paired Samples t-test
 - Item analysis of items that saw growth

Methodology

- Question 3: What learning opportunities in a methods course may help SUM?
 - Conducted interviews with four methods instructors who saw significant growth.
 - Asked about format and general philosophy of course
 - Asked about learning opportunities that may have helped increase mathematical understanding

Data Analysis and Findings

- Question 1: What are the areas of strength and what are the areas of weakness in SUM as prospective elementary teachers enter their methods course?

Conducted an item analysis on 11 items with highest number of correct answers and 11 items with lowest number of correct answers

Areas of Strength

- Six items from NOCK
 - Five of these common content knowledge
 - One was specialized content knowledge – representing fraction subtraction
- Five items from Geometry
 - Analyze characteristics of two and three dimensional shapes
 - Interpreting definitions of three dimensional shapes

Areas of Weakness

- NOCK – 9 items
 - One was common content knowledge $-x^y$
 - Eight were specialized content knowledge
 - Providing mathematical explanations (3)
 - Representing mathematical ideas (2)
 - Interpreting non-standard algorithms (3)
- Geometry – 2 items
 - Relationship between area and pi
 - Effects of changing one dimension on the area, volume and surface area

Indicators

- Previous content courses
- Do students who take math for teachers I and II score differently than those who do not? Yes $p=.008$, effect size .40

	\bar{x}	N	M
	0	8	3
	0	8	7

		F	S	t	d	
\bar{x}	\bar{x}		0		2	6
	\bar{x}					5

Quantity or Type

- Do students who take a higher number of content courses score differently? No, $p=.138$

			N
b	1	2	9
B	2	3	8
	3	4	8

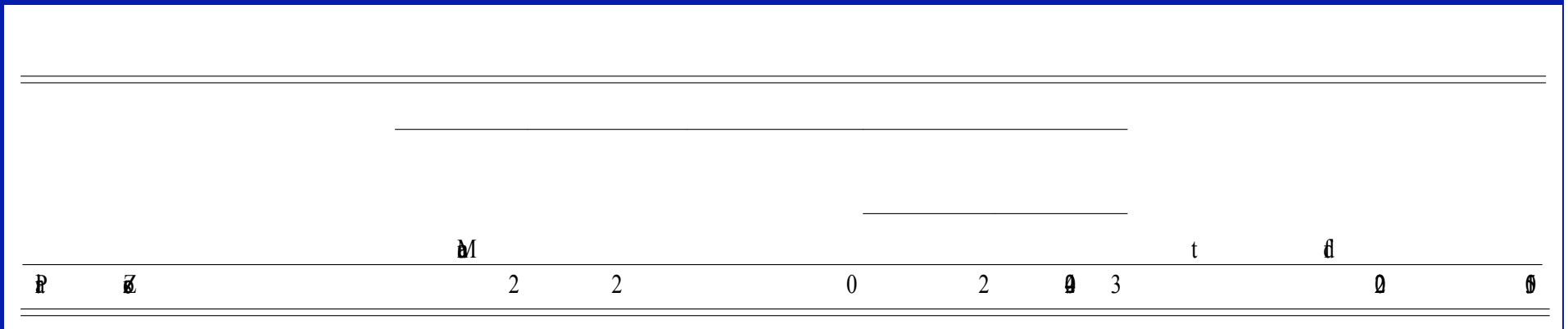
D			
b	MI	N	
2		4	9
3		8	8
4		2	8
B			4

e					
D					
b	a	d	MI	F	S
6		2	9		8
I		1			
H		2			
E		4			
B	0	4			
6		4			

a R

Question 2: Growth during Methods Course?

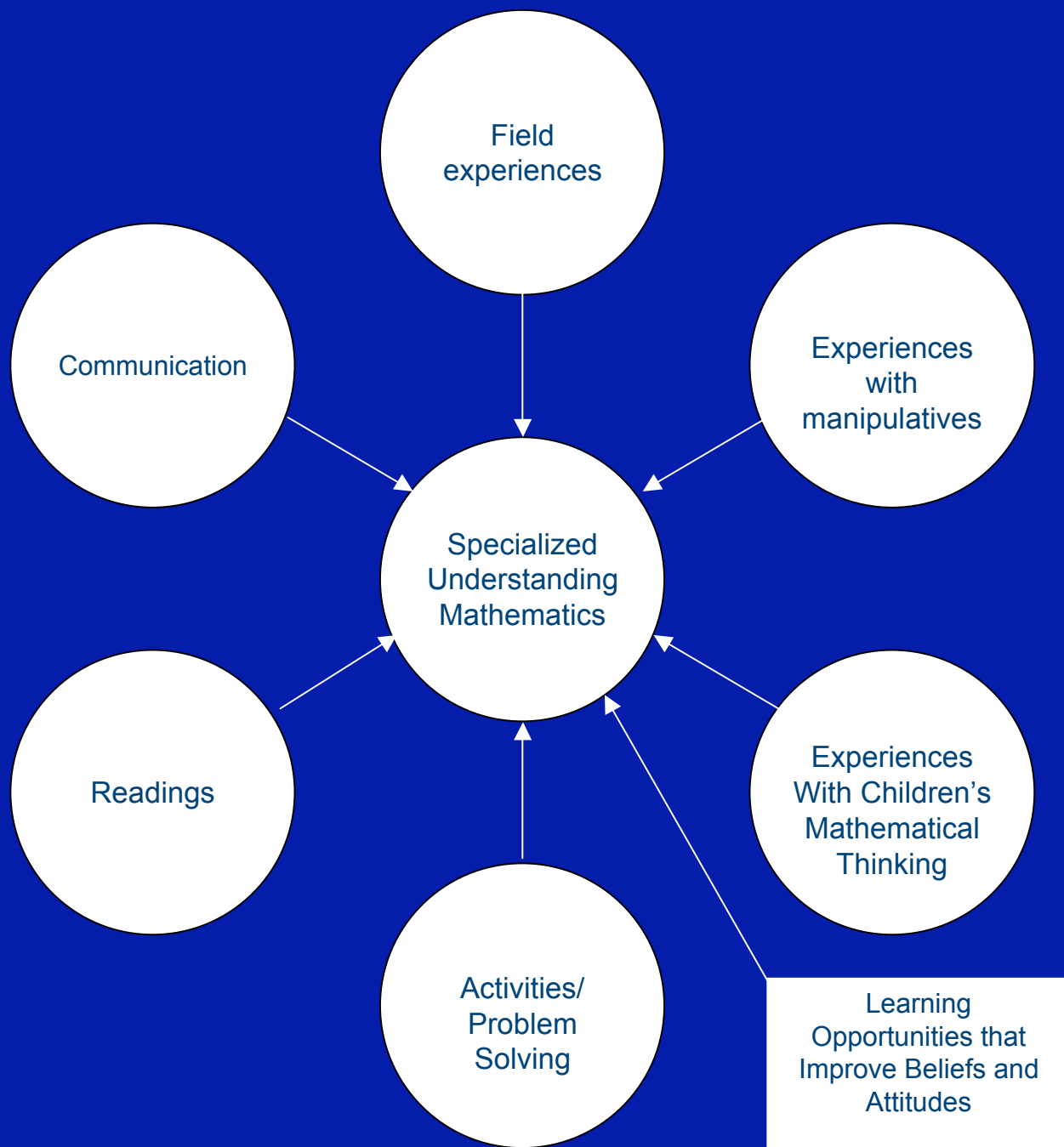
- Statistically significant growth was found
- Growth equivalent to about one item out of 48, $p = .015$, effect size = .123



Items with largest growth

- Four of the items that showed the most improvement were from the geometry content area
- Four were from the number and operation content area
- Of the four number and operation items that showed the most improvement, three of those were from the specialized content knowledge domain.

Question 3: What learning opportunities may help SUM?



Reading Opportunities

- Journal articles
- Textbooks
- Math curriculum materials
- Standards
- Children's Literature

Math Activities and Problem Solving Opportunities

- Construct their own knowledge
- Gain visual images
- Situated in a classroom setting, or the idea is related to children's thinking and pedagogical issues

Experiencing children's mathematical thinking opportunities

- Video clips of mathematics interviews with children
- Interviews with children
 - opportunities to listen to children talk and think about mathematics
 - experiences in forming good questions to encourage their thinking and to better understand their thinking
- Field experiences (well designed)
- Student work samples analysis

Manipulative Opportunities

- Provide “visual images” of the mathematics
- Help prospective teachers to make sense of the mathematics
- One instructor talked about how towards the end of the semester, the students do not pull the manipulatives off the cart as often as they are able to visualize them. They are still thinking with the visual images of the manipulatives but no longer feel as much of a need to actually use them once they understand the mathematics in that way.

Field Experience Opportunities

- Opportunities to improve SUM along with beliefs and attitudes about mathematics.
- Seeing a topic being taught in elementary classroom can lead to discussions on that topic in methods course
- Opportunities to see the depth of the mathematical thinking that the children are capable of and therefore help the prospective teachers to understand the need to learn mathematics more deeply themselves.

Communication Opportunities

- Using precise language about mathematics.
- Asking appropriate questions
- Listening to mathematical communications
- Providing explanations

Beliefs and Attitudes

- Affective goals are intertwined with content goals in these methods courses
- Improving beliefs and attitudes helps content knowledge, improving content knowledge helps beliefs and attitudes.
- While this study makes no claims about what learning opportunities may improve beliefs and attitudes, this researcher suspects that the six opportunities in this model would be a good theory to be tested.

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