

MATHEMATICAL HABITS OF MIND FOR TEACHING

FOCUS ON MATHEMATICS, PHASE II

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Slides available at
focusonmath.org

WE HAVE 45 MINUTES

“The trouble with talking too fast is you may say something you haven’t thought of yet.”

—Ann Landers

QUESTION FOR TODAY

What are the mathematical habits of mind that expert high school teachers use in their work?

OUTLINE

- 1 INTRODUCTION
- 2 FOCUS ON MATHEMATICS PHASE II
- 3 MATHEMATICAL HABITS OF MIND
- 4 THE FOM II RESEARCH STUDY

WORK TO DATE: *Focus on Mathematics*

- A targeted partnership, established in 2003
- Rooted in *PROMYS for Teachers*, initiated in 1989

Details are at the Poster Session

WORK TO DATE: *Focus on Mathematics* GOALS

- To help secondary teachers put mathematics at the core of their profession
- To establish a *mathematical community* in which teachers, mathematicians, and mathematics educators work together, doing mathematics

Details are at the Poster Session

WORK TO DATE: *Focus on Mathematics* PROGRAMS

The FoM partnership has established

- Study groups
- Seminars and colloquia
- A new graduate degree:
 - Master of Mathematics for Teaching (MMT)
- Summer institutes
- Mathematics fairs and Mathematics Expo
- Research collaboratives
- Evaluation and case studies

Details are at the Poster Session

WORK TO DATE: *Focus on Mathematics*

CASE STUDY RESEARCH

The case study research was designed to document and describe 6 teachers' experiences in the MMT program.

Research Questions

- 1 What did teachers learn?
- 2 How did teachers' beliefs about teaching and learning mathematics change?
- 3 What critical experiences informed these changes?
- 4 How did teachers change their classroom practice?

IMPLEMENTATION AND RESEARCH

Programs

- Support the existing community
 - to carry on its work
 - to provide a context for the research
- Develop a long-range research program
- Conduct a focussed research study

THE RESEARCH PROGRAM

One part of the work of *FOM-II* will be to develop, in collaboration with others working in this area, a long-term research program to address the question:

What are the connections between secondary teachers' mathematical knowledge for teaching and secondary students' mathematical understanding and achievement?

The first step in this design will be to convene a national conference at IM&E in 2011 to showcase existing work and to formulate some coherent directions for future work

THE RESEARCH STUDY

The second major research effort of *FOM-II* is a focussed study, now underway, that will contribute to the larger research program. It will study the question:

What are the mathematical habits of mind that high school teachers use in their professional lives and how can we measure them?

THE RESEARCH STUDY

The plan:

- 1 Develop a description and provide examples of mathematical habits of mind that are useful in high school teaching
- 2 Develop a paper and pencil instrument to assess these mathematical habits
- 3 Develop a classroom observation protocol to do the same
- 4 Calibrate the instruments in items (2) and (3) above

MATHEMATICAL HABITS OF MIND

What are *Mathematical Habits of Mind*?

- Specialized ways of approaching mathematical problems and thinking about mathematical concepts that **resemble the ways employed by mathematicians**.
- Examples of such habits include:
 - performing thought experiments,
 - using mathematical language precisely,
 - abstracting regularity from repeated calculations,
 - seeking structural similarities,
 - reasoning by continuity,
 - “chunking,”
 - delayed evaluation.

MATHEMATICAL KNOWLEDGE FOR TEACHING

The notion of mathematical habits of mind is a critical component of *mathematical knowledge for teaching*.

- MKT is the knowledge used to carry out *the work of teaching mathematics*.
- Examples of MKT include:
 - explaining terms and concepts to students,
 - interpreting students' statements and solutions,
 - judging textbook treatments of particular topics,
 - using representations accurately in the classroom.

MATHEMATICAL KNOWLEDGE FOR TEACHING

Work with secondary teachers uncovers some other aspects.

- using mathematics to design tasks for students,
- sequencing lessons to bring coherence to seemingly disparate topics,
- “layering” mathematics to separate convention from substance,
- situating secondary mathematics in the larger landscape of the discipline.

MATHEMATICAL HABITS OF MIND AND MKT

How do mathematical habits of mind fit into the theory of mathematical knowledge for teaching?

Expert teachers know mathematics in several different ways:

- *They know mathematics as a scholar:* They have a solid grounding in classical mathematics, including its major results, its history of ideas, and its connections to precollege mathematics.
- *They know mathematics as an educator:* They understand the thinking that underlies major branches of mathematics and how this thinking develops in learners.

MATHEMATICAL HABITS OF MIND AND MKT

- \Rightarrow *They know mathematics as a mathematician:* They have experienced a sustained immersion in mathematics that includes performing experiments and grappling with problems, building abstractions from the experiments, and developing theories that bring coherence to the abstractions.
- \Rightarrow *They know mathematics as a teacher:* They are expert in uses of mathematics that are specific to the profession, including the ability “to think deeply about simple things” (*Arnold Ross*), the craft of task design, and the “mining” of student ideas.

MATHEMATICAL HABITS OF MIND AND MKT

How do mathematical habits of mind fit into the theory of mathematical knowledge for teaching?

Hypothesis: The ability to think in characteristically mathematical ways helps secondary teachers:

- make sense of students' half-formed insights,
- find underlying connections among “different” topics,
- conduct classrooms that are more textured and less flat,
- encourage students to struggle with hard ideas,
- develop student tasks that uncover mathematical thinking,
- help students see the importance and utility of mathematical habits of mind.

RESEARCH TEAM

The research study will be carried out by a team of mathematicians and mathematics educators from BU and EDC:

- Suzanne Chapin (BU; co-PI, advisor)
- Al Cuoco (EDC; co-PI)
- Wayne Harvey (EDC; advisor)
- Ryota Matsuura (BU; co-director)
- Barbara Scott Nelson (EDC; advisor)
- Glenn Stevens (BU; PI)
- Sarah Sword (EDC; co-director)

RESEARCH GOAL

- **Development of two assessment instruments:**
 - Paper and pencil assessment of mathematical habits of mind for teaching.
 - Classroom video observation protocol for mathematical habits of mind that teachers exhibit in their instruction.
- Our instruments will be modeled after the work of D. Ball and H. Hill for documenting *mathematical knowledge for teaching* in elementary school teachers.
- We chose to focus on high school teachers because our ideas about MHoM were developed in the context of our work with them.

RESEARCH DESIGN

The research study will:

- Develop a detailed definition and categorization of *mathematical habits of mind for teaching* (MHoM-T).
- Develop and pilot-test a **paper and pencil assessment** that measures the nature and degree of MHoM-T.
- Develop and pilot-test an **observation protocol** for MHoM as exhibited in high school mathematics instruction.
- Conduct reliability and validity tests of both instruments to determine the relationship between performance on the written test and a focus on MHoM in classroom instruction.

ADDITIONAL NOTES ON THE STUDY

- Our project will be conducted over three years (2009–2012).
- **Disclaimer:** The development and validation of a truly reliable paper and pencil assessment is beyond the scope of our research project. Our instruments will be *provisional* in this sense and provide data for the long-term research program.

PROJECT DETAILS

Discussion of the proposed plans for:

- Research Sample
- Instrument Development
- Data Collection
- Data Analysis

RESEARCH SAMPLE (OVERVIEW)

We will work with three groups of teachers to develop our assessment instruments:

- In Year 1 (2009 – 2010), we will pilot sample questions for the paper and pencil assessment with approximately 30 alumni teachers from *PROMYS for Teachers* (PfT) program.
- We will select two cohorts of six teachers each for classroom observation, videotaping, and interviewing.

RESEARCH SAMPLE (COHORT 1)

Cohort 1 teachers will help us identify precisely those mathematical habits of mind that high school teachers actually use in their work with students.

- These six teachers will be selected from the PfT communities that we have developed over the last 10 years.
- Teachers with a broad sampling of strengths in MHoM-T:
 - using mathematical language precisely,
 - designing mathematical experiments,
 - generalizing from concrete examples.
- The observations/interviews of these purposively selected teachers will give us **rich source of data for developing our assessment instruments.**

RESEARCH SAMPLE (COHORT 2)

Cohort 2 teachers:

- These six teachers will be selected from the approximately 35 participants in the PfT (2010) program.
- Cohort 2 will **serve as a sample for the pilot-testing and field-testing of the observation protocol.**
- Three with high levels of MHoM-T and three with low levels, as measured by the paper and pencil assessment to test the observation protocol's capacity to identify varying degrees of MHoM in classroom instruction.

INSTRUMENT DEVELOPMENT

PAPER AND PENCIL ASSESSMENT

We will develop a collection of problems that measure the MHoM that high school teachers use in their profession.

- Cover a range of mathematical topics in grades 9–12.
- Contain short-answer and/or multiple-choice questions.
- Can be completed in one hour.
- Each question may require the use of one or more MHoM-T.

INSTRUMENT DEVELOPMENT

PAPER AND PENCIL ASSESSMENT

Pilot-testing of the paper and pencil assessment will be conducted as follows:

- Sample paper and pencil questions will be given to alumni teachers from the PfT program.
- “Think aloud” interviews will be conducted with mathematicians and mathematics educators to test whether or not the assessment questions measure the particular habits of minds we claim they do.

SAMPLE PAPER AND PENCIL PROBLEM

One MHoM-T of interest is:

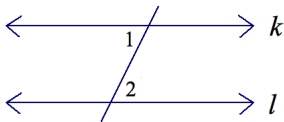
using mathematical language precisely.

Characteristics of this habit include:

- Giving a thorough and complete explanation of a process,
- Inventing and using notations,
- Describing observed phenomena in general terms,
- **Understanding and using logical implications.**

SAMPLE PAPER AND PENCIL PROBLEM

Students are looking at the following diagram.



Here's the ensuing conversation:

Al: If lines k and l are parallel, then $m(\angle 1) = m(\angle 2)$.

Betty: If $m(\angle 1) = m(\angle 2)$, then lines k and l are parallel.

Carol: What's the difference? They're both correct, aren't they? Aren't they saying the same exact thing in two different ways?

SAMPLE PAPER AND PENCIL PROBLEM

If you are the teacher, which of the following examples would you use to help Carol see the *distinction* between Al and Betty's statements?

- (A) If it's 10 AM in New York City, then it's 7 AM in Los Angeles.
- (B) If I live in Boston, then I live in Massachusetts.
- (C) A *transversal* is a line that cuts across two or more lines.

SAMPLE PAPER AND PENCIL PROBLEM

In this scenario, the teacher must explain the *distinction* between a conditional statement (i.e. if-then) and its converse.

- Choice (c) introduces vocabulary – it is least relevant to the task with which the teacher faces.
- Choice (a) is a conditional statement. But both the statement and its converse are true, so it is insufficient for clarifying Carol's confusion.
- **Choice (b)** is a true conditional statement whose converse (i.e. "If I live in Massachusetts, then I live in Boston") is false. **It is an optimal example for the teacher's task.**

ANOTHER SAMPLE PAPER AND PENCIL PROBLEM

Another MHoM-T of interest is:

**getting to the heart of the matter
(aka using abstraction).**

Characteristics of this habit include:

- looking beneath the surface features of a situation for mathematical structure,
- seeing general results in specific contexts,
- finding regularity in repeated actions,
- **Using definitions in different domains.**

ANOTHER SAMPLE PAPER AND PENCIL PROBLEM

Let E be the set of even numbers:

$$E = \{ \dots - 6, -4, -2, 0, 2, 4, 6, 8 \dots \}$$

Consider the following three elements if E :

I: 2

II: 10

III: 20

Which of these are prime in E ?

- 1 I only
- 2 II
- 3 III only
- 4 I and II only
- 5 II and III only
- 6 None of the above

ANOTHER SAMPLE PAPER AND PENCIL PROBLEM

In this scenario, one must decide what it means to be prime in a system different from the ordinary rational integers. In \mathbb{Z} , an integer is prime if and only if it is the product of two other integers, neither of which is a unit.

Note: Teachers might use this example to show systems that do not have unique factorization into primes: 6^2 and $18 \cdot 2$ are different prime factorizations of 36 in E .

GOT TIME?

- If so, we can try one and compare it with our preliminary pilot results.
- If not, you can take it with you and play with it on the flight home.

INSTRUMENT DEVELOPMENT

CLASSROOM VIDEO OBSERVATION PROTOCOL

During Year 1 (2009 – 2010):

- Classroom observations/videotapes of Cohort 1 teachers will be used to document categories of teacher and student behaviors/interactions that reflect various MHoM-T.
- Provisional coding scheme and rubric will be developed to measure how often and in what manner each habit is used.

INSTRUMENT DEVELOPMENT

CLASSROOM VIDEO OBSERVATION PROTOCOL

During Year 2 (2010 – 2011), we will continue to revise and pilot-test the observation protocol.

- Observation/videotaping of Cohort 2 teachers; two consecutive classes for each teacher.
(Recall: 3 with high levels MHoM-T; 3 with low levels.)
- Review of videotapes (of both Cohorts 1 and 2) and the observation protocol by mathematicians and mathematics educators.

INSTRUMENT DEVELOPMENT

CALIBRATION OF INSTRUMENTS

Our goal is to determine if the paper and pencil assessment predicts practice, as measured by the observation protocol.

- Mathematics education graduate students will be trained to use coding scheme and the rubric in the observation protocol.
- They will code the videotapes of Cohort 2 teachers, while being “blind” to the MHoM-T status of these teachers.
- Multiple graduate students will code the same teacher’s videotape to establish inter-rater reliability of the coding scheme.

DATA COLLECTION

- During Years 1, 2, and 3, we will field-test the paper and pencil assessment by administering it to the PfT participants as a pre- and post-test.

Note: These teachers will be distinct from those who will participate in the development of the instrument.

- Year 1 teachers: Two equal-sized subgroups, with an even distribution of MHoM-T (as measured by their pre-tests).
 - One subgroup will take the post-test immediately after their PfT mathematical immersion experience.
 - Other subgroup will take the post-test one year later, to measure change in their MHoM-T over time.

DATA ANALYSIS

PAPER AND PENCIL ASSESSMENT

- Each problem on the paper and pencil assessment will be scored out of 5 points, depending on the level of MHoM-T exhibited by the teacher's response.
- For each PfT participant taking the pre- and post-tests, the normalized gain (G) will be calculated.
- Gain scores will be correlated with other variables:
 - mathematical background,
 - teaching experience,
 - participation in professional development activities.

DATA ANALYSIS

CLASSROOM VIDEO OBSERVATION PROTOCOL

- During Year 3 (2011 – 2012), videotapes of Cohort 2 teachers will be recoded using the “final” observation protocol.
- For each mathematical habit listed in the coding scheme, the rubric will provide a rating scale (ranging from 0 to 5) for the occurrence of this habit in the teacher’s instruction.
- The data will be analyzed to correlate the mathematical habits exhibited in the classroom with MHoM-T as measured by the paper and pencil assessment.

APPLICATIONS

- The assessment instruments will be used to study the effects of mathematical immersion experiences (such as the PfT program) on teachers' development of MHoM-T and how they apply these habits in their professional lives.

Note: Not meant to be used for evaluation of teachers.

- Results of these assessment instruments will inform decisions on the preparation of pre-service teachers and professional development for in-service teachers.

PROGRESS TO DATE

Work done since September, 2009:

- Defining and categorizing MHoM-T
- Draft of paper and pencil assessment (~ 50 problems).
- Administered P & P to PfT 2009 participants as pre/post.
- Informal observations in local high school classrooms.

AND NOW...

- Questions, comments, heckles, ... ?
- Ideas about the overarching research question?
- **Advice and suggestions?**

FOR MORE INFORMATION

- **About *FoM-II*:**
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FOR MORE INFORMATION

- About the case studies:

- pdfs:



- www.focusonmath.org/FOM/PERG/Introduction.pdf



- www.focusonmath.org/FOM/PERG/Methodology.pdf



- www.focusonmath.org/FOM/PERG/hsmathteacher.pdf

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Thanks